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A new criteria for the update speed needed for the correct real-time simulation of the continuous dynamics of automotive systems.

Summary

A new criteria is derived for the update frequency needed to simulate the dynamics of systems represented by coupled ordinary nonlinear differential equations. The frequency response of the Euler Numerical Integration Rule is computed and compared to that of an ideal integrator. The result is a table relating the ratio of simulation update frequency to the highest frequency being simulated versus deviation. For example, for less than a 0.1 db deviation from an ideal integrator's frequency response the simulation should run at 12.8 times the highest frequency. This conclusion dictates the computer power needed.

INTRODUCTION

The real-time simulation of the dynamics of automotive systems is finding wide application throughout the industry. A common application is hardware-in-the-loop (HIL) simulation where mixtures of real and simulated components interact in real time in order to test the components as well as the overall behavior of the automobile. Examples include the simulation of vehicle dynamics including tire-road interaction to test antilock braking systems (ABS) or the simulation of engine and drive train dynamics to test fuel-injection electronic control units. The benefits include faster time to market and the reduction of costly testing on the test track with real vehicles. XANALOG's XANALOG/RT Series of real-time hardware-in-the-loop and controller prototyping systems are currently in use for testing ABS, fuel injection control, cruise control, transmission control and other automotive systems in Europe, Japan and the United States. In working with these users as well as users in the aerospace and industrial automation industries the question often arises as to how fast a simulation must be updated given the highest frequency components present in the dynamics of the mechanism being simulated. For example, if the simulation must handle frequencies up to 100 Hz, with what frequency must the simulation be computed? When this question was posed to experienced simulationists their responses ranged from 10 to 20 times the highest frequency that the simulation needs to represent. Neither the simulationists or the literature had a derivation from first principles of why 10 to 20 times was good value with which to work. This paper starting from first principles shows that the experienced simulationists were correct. The derivation which follows computes the frequency response of Euler integration and compares it to the frequency response of an ideal integrator. The result is a table which relates the ratio of update frequency to highest frequency in the simulation to deviation from ideal integration for the case of the Euler Integration Rule.

DISCUSSION

DERIVATION OF THE FREQUENCY RESPONSE OF EULER INTEGRATION

The frequency response of the Z-Domain circuit of Figure 1 which is equivalent to the Euler Integration Rule can be computed as follows:

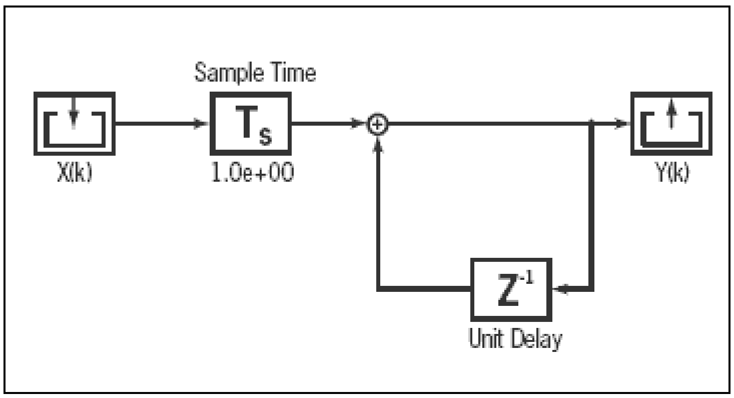

Figure 1. A Z-Domain circuit equivalent to Euler integration.

The Euler approximation to the area, Y(N), under the curve x(t) is given by:

$$Y(N) = \sum_{n=1}^{N} T_s x(n)$$

where T_s is the time between the samples of x(t) and N is the number of samples.

$$Y(N+1) = T_s x(N+1) + \sum_{n=1}^{N} T_s x(n)$$

$$Y(N+1) = T_s x(N+1) + Y(N)$$

or letting n=N+1:

$$Y(n) = T_s x(n) + Y(n-1)$$

Now transforming to the Z domain:

$$Y(z) = T_s X(z) + z^{-1} Y(z) \quad \text{or}$$

$$\frac{Y(z)}{X(z)} = H_E(z) = \frac{T_s}{1-z^{-1}}$$

where H_E (z) is the transfer function of the Euler integrator.

Now let $z = e^{j\alpha}$ where $\alpha = 2\pi(\omega/\omega_s)$ and $\omega_s = 2\pi/T_s$:

$$H_E(e^{j\alpha}) = \frac{2\pi}{w_s(1-e^{-j\alpha})}$$

using $1-e^{j\alpha} = e^{j\alpha/2}(e^{-j\alpha/2} - e^{j\alpha/2})$

and then $-2j\sin(\theta) = e^{-j\theta} - e^{j\theta}$

$$H_E(e^{j\alpha}) = \frac{2\pi}{-(2j\omega_s)e^{-j\alpha/2}SIN(\frac{\alpha}{2})}$$

Find the magnitude given $|e^{-j\alpha/2}| = 1$ and following the usual convention of substituting w for the $e^{j\alpha}$ which appears as the argument of H_E.

$$|H_E(\omega)| = \frac{1}{|\frac{\omega_s}{\pi}| \, |\sin(\pi\frac{\omega}{\omega_s})|}$$

note for small θ: $\sin\theta = \theta$ thus

$$H_E(\omega) = \frac{1}{|\omega|}$$

COMPARISON OF EULER INTEGRATION TO IDEAL INTEGRATION

An ideal integrator has the Laplace domain transfer function: $H(s) = 1/s$ or for response to complex exponential yields, the magnitude of the transfer function $|H_I(\omega)| = |1/\omega|$. Thus, the question reduces to the following: over what range does $(\omega/\pi)\sin(\pi\omega/\omega_s$ behave like ω. The two functions are plotted in Figure 2 for $\omega_s = 1$. The magnitude's functions are tabulated in Table 1.

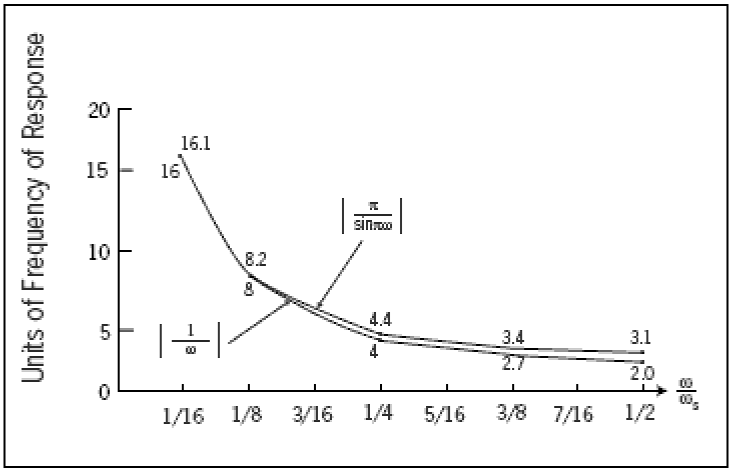

Figure 2. Plots of the magnitudes of the frequency responses of Euler integration and an ideal integrator.

For high-fidelity simulation, one could insist that the Euler approximation deviate from the ideal integration by no more than 0.1 db. Table 1 shows that an error of 0.1 db occurs when the sampling frequency is 12.8 times the highest frequency in the signal being integrated.

$\frac{\omega_s}{\omega}$	Magnitude of Euler's frequency response	Magnitude of ideal integrator's frequency response	Euler's deviation from ideal
64	64.02	64.00	0.0003 db
32	32.05	32.00	0.001 db
16	16.10	16.00	0.005 db
12.8	12.93	12.80	0.09 db
8	8.209	8.00	0.2 db
4	4.442	4.00	0.91 db

Table 1. Euler's deviation from ideal for sampling frequency/highest frequency being integrated.

CONCLUSION

This paper derives from first principles a table which relates a ratio (the highest frequency in a simulation to the update rate of the simulation) with the deviation from ideal integration for the case of Euler integration. This ratio has major economic implications in specifying a simulation system.

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