

XANALOG.COM Presents:

Using XANALOG block diagrams to express differential equations



Block diagrams are familiar to many scientists and engineers working with dynamic systems, but there are others who prefer differential equations. This note hopes to bridge this gap by showing how to convert differential equations to block diagrams.

Combining the two block diagrams in Figure 1 yields the block diagram of Figure 2. Figure 2 was constructed on a CRT with the XANALOG block diagram editor. The block diagram can be compiled with XANALOG block diagram compilers and used to compute the dynamic response of the system. Responses can be computed by exciting the x input or entering initial conditions for y and y' . Note that the label under each integrator represents the output of the integrator.

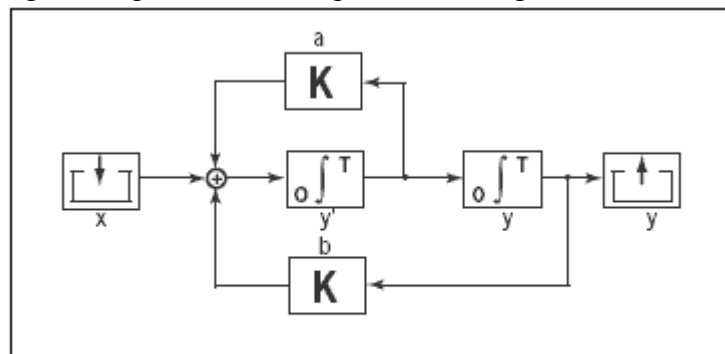


Figure 2. Second order differential equation in XANALOG block diagram form.

Consider the following second-order differential equation:

$$\frac{d^2}{dt^2} y(t) - a \frac{d}{dt} y(t) - by(t) = x(t) \quad (1)$$

It can be written as:

$$y'' - ay' - by = x \quad (2)$$

Rearranging the terms we obtain:

$$x + ay' + by = y'' \quad (3)$$

The left side of Equation (3) is shown in block diagram form in the left side of Figure 1. The y'' from the right side of Equation (3) is integrated twice yielding y' and y as shown in the right side of Figure 1.

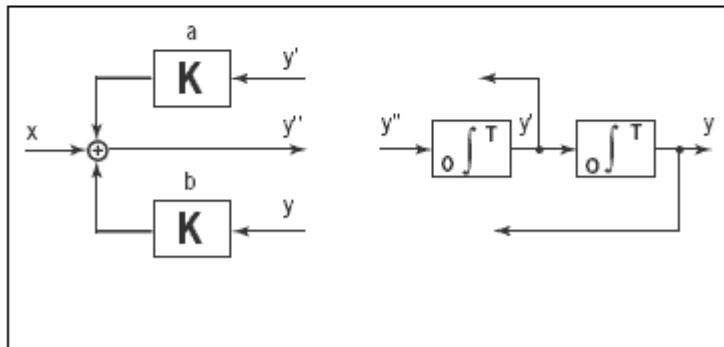


Figure 1. The left side of Equation (3) and the expanded right side of Equation (3) in block diagram form.

A PROCEDURE FOR CONVERTING

DIFFERENTIAL EQUATIONS TO BLOCK DIAGRAMS

The process illustrated in Figures 1 and 2 forms the basis for a procedure that can be applied as follows:

1. Determine the highest derivative present in the differential equation and construct a block diagram with the same number of integrators as the order of the highest derivative. Connect the integrators to one another in a line from left to right. See the right side of Figure 1.
2. Take the undifferentiated variable and label the output of the integrator on the right with it. Then enter the various derivatives values ending with the highest derivatives as the input to the left-most integrator. This is shown in the right side of Figure 1.
3. Connect a summer to the input of the left-most integrator. See the left side of Figure 1.
4. Rewrite the differential equation so that the highest derivative is expressed as a sum of the lower derivatives, the undifferentiated variable and the inputs. See Equation (3).
5. Connect the weighted variables to the summer according to the equation derived in step (4) as shown in the left side of Figure 1.

This procedure is useful with a broad class of differential equations. Algebraic, transcendental, or tabulated functions could replace the constant coefficients of the example given here.